LIBERTY PAPER SET

STD. 12 : Mathematics

Full Solution

Time : 3 Hours **ASSIGNMENT PAPER 9**

PART A

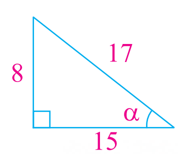
1. (D) **2.** (B) **3.** (C) **4.** (B) **5.** (D) **6.** (B) **7.** (B) **8.** (A) **9.** (B) **10.** (B) **11.** (A) **12.** (D) **13.** (B) **14.** (A) **15.** (A) **16.** (A) **17.** (C) **18.** (D) **19.** (C) **20.** (B) **21.** (A) **22.** (A) **23.** (C) **24.** (B) **25.** (D) **26.** (B) **27.** (D) **28.** (C) **29.** (A) **30.** (B) **31.** (A) **32.** (B) **33.** (D) **34.** (B) **35.** (C) **36.** (B,C) **37.** (A) **38.** (D) **39.** (C) **40.** (A) **41.** (B) **42.** (C) **43.** (A) **44.** (D) **45.** (A) **46.** (D) **47.** (C) **48.** (D) **49.** (C) **50.** (C)

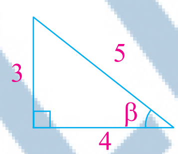
PART B

SECTION A

1.

L.H.S. $= sin^{-1}\dfrac{8}{17} + sin^{-1}\dfrac{3}{5}$





$\alpha = sin^{-1}\dfrac{8}{17} = cos^{-1}\dfrac{15}{17}$ | $\beta = sin^{-1}\dfrac{3}{5} = cos^{-1}\dfrac{4}{5}$

$\therefore sin\ \alpha = \dfrac{8}{17}, cos\ \alpha = \dfrac{15}{17}$ | $sin\ \beta = \dfrac{3}{5}, cos\ \beta = \dfrac{4}{5}$

$\therefore tan\ \alpha = \dfrac{8}{15}$ | $tan\ \beta = \dfrac{3}{4}$

Here, $\alpha + \beta = ?$

$$tan\ (\alpha + \beta) = \frac{tan\ \alpha + tan\ \beta}{1 - tan\ \alpha\ tan\ \beta}$$

$$= \frac{\frac{8}{15} + \frac{3}{4}}{1 - \frac{8}{15}\cdot\frac{3}{4}}$$

$$= \frac{\frac{32+45}{60}}{\frac{60-24}{60}}$$

$$tan\ (\alpha + \beta) = \frac{77}{36}$$

$$\therefore \quad \alpha + \beta = tan^{-1}\left(\frac{77}{36}\right)$$

$$\therefore sin^{-1}\frac{8}{17} + sin^{-1}\frac{3}{5} = tan^{-1}\frac{77}{36}$$

2.

$tan^{-1}\left(\dfrac{x}{\sqrt{a^2 - x^2}}\right)$

Suppose, $x = a\ sin\ \theta$

$\therefore\ sin\theta = \dfrac{x}{a}$

$\therefore\ \theta = sin^{-1}\dfrac{x}{a},\ \theta \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

$= tan^{-1}\left(\dfrac{a\,sin\theta}{\sqrt{a^2 - a^2 sin^2\theta}}\right)$

$= tan^{-1}\left(\dfrac{\not{a}\ sin\theta}{\not{a}\sqrt{1 - sin^2\theta}}\right)$

$= tan^{-1}\left(\dfrac{sin\ \theta}{cos\ \theta}\right)$

$= tan^{-1}\ (tan\ \theta)$

$= \theta$

$= sin^{-1}\dfrac{x}{a}$

$|x| < a$

$\therefore\ -a < x < a$

$\therefore\ -1 < \dfrac{x}{a} < 1$

$\therefore\ sin\left(-\dfrac{\pi}{2}\right) < sin\,\theta < sin\,\dfrac{\pi}{2}$

$\therefore\ -\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$

$\theta \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right) \subset \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

$\therefore\ cos\ \theta > 0$

3.

$sin\ x + sin\ y = tan\ xy$

Differentiate w.r.t. x,

$\therefore\ \dfrac{d}{dx}(sin\ x + sin\ y) = \dfrac{d}{dx}(tan\ xy)$

$\therefore\ cos\ x + cos\ y\dfrac{dy}{dx} = sec^2\ xy\left(x\dfrac{dy}{dx} + y(1)\right)$

$\therefore\ cos\ x + cos\ y\dfrac{dy}{dx} = x\ sec^2\ xy\dfrac{dy}{dx} + y\ sec^2\ xy$

$$\therefore \quad \cos y \frac{dy}{dx} - x \sec^2 xy \frac{dy}{dx} = y \sec^2 xy - \cos x$$

$$\therefore \quad \frac{dy}{dx}(\cos y - x \sec^2 xy) = y \sec^2 xy - \cos x$$

$$\therefore \quad \frac{dy}{dx} = \frac{y \sec^2 xy - \cos x}{\cos y - x \sec^2 xy}$$

4.

Take, $\tan \frac{x}{2} = t$

$$\therefore \quad \sec^2 \frac{x}{2} \cdot \frac{1}{2} dx = dt$$

$$\therefore \quad dx = \frac{2\,dt}{1+t^2}$$

$$x \to 0 \Rightarrow t \to 0$$

$$x \to \frac{\pi}{2} \Rightarrow t \to 1$$

$$= \int_0^1 \frac{\frac{2\,dt}{1+t^2}}{3 + \frac{2(1-t^2)}{1+t^2}}$$

$$= \int_0^1 \frac{2\,dt}{3+3t^2+2-2t^2}$$

$$= 2\int_0^1 \frac{1}{t^2+5}\,dt$$

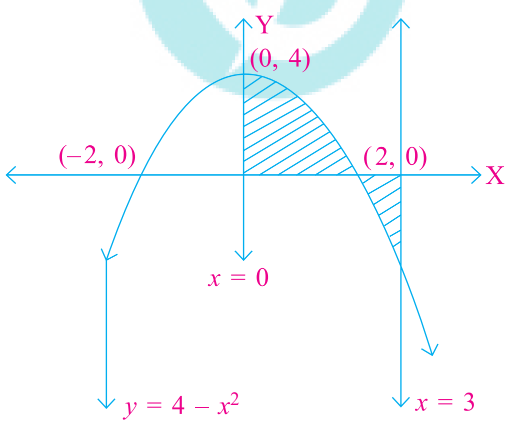
$$= 2\int_0^1 \frac{1}{t^2+(\sqrt{5})^2}\,dt$$

$$= 2\left[\frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{t}{\sqrt{5}}\right)\right]_0^1$$

$$= \frac{2}{\sqrt{5}}\left\{\tan^{-1}\left(\frac{1}{\sqrt{5}}\right) - \tan^{-1}\left(\frac{0}{\sqrt{5}}\right)\right\}$$

$$= \frac{2}{\sqrt{5}} \tan^{-1} \frac{1}{\sqrt{5}}$$

5.



$$I = \int_0^2 y\,dx + \left|\int_2^3 y\,dx\right|$$

$$= \int_0^2 (4-x^2)\,dx + \left|\int_2^3 (4-x^2)\,dx\right|$$

$$= \left[4x - \frac{x^3}{3}\right]_0^2 + \left|\left(4x - \frac{x^3}{3}\right)_2^3\right|$$

$$= 8 - \frac{8}{3} + \left|\left(12 - \frac{27}{3}\right) - \left(8 - \frac{8}{3}\right)\right|$$

$$= \frac{16}{3} + \left|3 - \frac{16}{3}\right|$$

$$= \frac{16}{3} + \left|\frac{9-16}{3}\right|$$

$$= \frac{16}{3} + \frac{7}{3}$$

$$= \frac{23}{3}$$

$$A = |I|$$

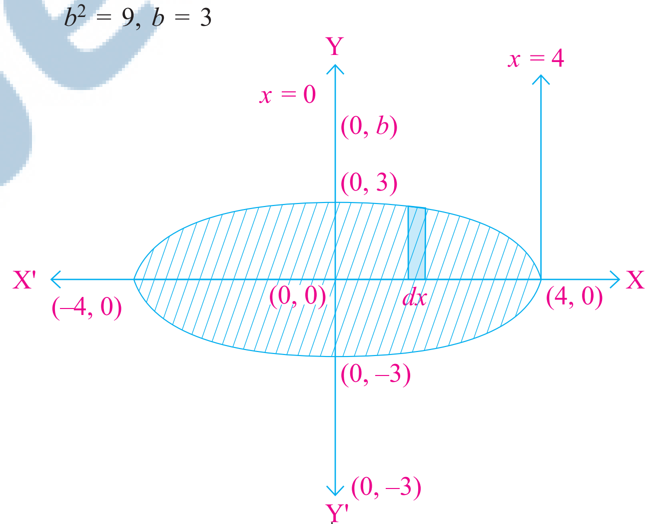
$$= \frac{23}{3} \text{ sq. unit}$$

6.

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

$a^2 = 16$, $a = 4$ $(a > b)$

$b^2 = 9$, $b = 3$



Required Area :

A = 4 × Area bounded in the first quadrant

$\therefore$ A = 4|I|

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

$$\therefore \quad y^2 = 9\left(1 - \frac{x^2}{16}\right)$$

$$\therefore \quad y^2 = \frac{9}{16}(16 - x^2)$$

$$\therefore \quad y^2 = \frac{3}{4}\sqrt{16-x^2}$$

$$I = \int_0^4 y\,dx$$

$$I = \int_0^4 \frac{3}{4}\sqrt{16-x^2}\,dx$$

$$I = \frac{3}{4}\int_0^4 \sqrt{16-x^2}\,dx$$

$$I = \frac{3}{4}\left[\frac{x}{2}\sqrt{16-x^2} + \frac{16}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_0^4$$

$$I = \frac{3}{4}\left[\left(\frac{4}{2}(0)+8\sin^{-1}(1)\right)-\left(0+\sin^{-1}(0)\right)\right]$$

$$I = \frac{3}{4}\left(8\cdot\frac{\pi}{2}\right)$$

$$I = 3\pi$$

Now, $A = 4|I|$

$= 4|3\pi|$

$\therefore \quad A = 12\pi$ sq. unit

7.

$$\frac{dy}{dx} + 3y = e^{-2x} \quad \ldots (1)$$

Compare given equation with $\frac{dy}{dx} + P(x)\,y = Q(x)$

$P(x) = 3$

$Q(x) = e^{-2x}$

Integrating factor I.F. $= e^{\int P(x)\,dx}$

$= e^{\int 3\,dx}$

$= e^{3x}$

Multiply equation (1) by e^{3x},

$\therefore\ e^{3x}\frac{dy}{dx} + 3y\,e^{3x} = e^{3x}\,e^{-2x}$

$\therefore\ e^{3x}\frac{dy}{dx} + 3y\,e^{3x} = e^{x}$

$\therefore\ \frac{d}{dx}(y\,e^{3x}) = e^{x}$

$\therefore\ y\,e^{3x} = \int e^{x}\,dx$

$\therefore\ y\,e^{3x} = e^{x} + c$

$\therefore\ y = e^{-2x} + ce^{-3x}$

Which is required general solution.

8.

The angle θ between two vectors $\vec{a}$ and $\vec{b}$ is given by,

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|}$$

Now, $\vec{a}\cdot\vec{b} = (\hat{i}+\hat{j}-\hat{k})\cdot(\hat{i}-\hat{j}+\hat{k})$

$= 1 - 1 - 1$

$= -1,\ |\vec{a}| = |\vec{b}| = \sqrt{3}$

$$\cos\theta = \frac{-1}{\sqrt{3}\cdot\sqrt{3}}$$

Therefore, we have $\cos\theta = -\frac{1}{3}$

Hence, the required angle is $\theta = \cos^{-1}\left(-\frac{1}{3}\right)$

OR $\theta = \pi - \cos^{-1}\left(\frac{1}{3}\right)$

9.

$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ is parallel to line

A line is parallel to vector $3\hat{i} + 2\hat{j} - 2\hat{k}$

$\therefore$ Vector equation of line

$\vec{b} = 3\hat{i} + 2\hat{j} - 2\hat{k}$

$\vec{r} = \vec{a} + \lambda\vec{b},\ \lambda \in R$

$\therefore\ \vec{r} = (\hat{i}+2\hat{j}+3\hat{k}) + \lambda(3\hat{i}+2\hat{j}-2\hat{k}),\ \lambda \in R$

Cartesian equation : $\frac{x-1}{3} = \frac{y-2}{2} = \frac{z-3}{-2}$

10.

Suppose, the makes an angle α, β and γ with X-axis, Y-axis and Z-axis.

$\alpha = \beta = \gamma$

Direction cosine of line are,

$\cos\alpha, \cos\beta, \cos\gamma$ Au.

$\therefore\ \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\therefore\ 3\cos^2\alpha = 1$

$\therefore\ \cos\alpha = \pm\frac{1}{\sqrt{3}}$

$\therefore\ \cos\alpha = \cos\beta = \cos\gamma = \pm\frac{1}{\sqrt{3}}$

$\therefore$ Direction cosine of line $= \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$

11.

Let K denote the event that the card drawn is king and A be the event that the card drawn is an ace. Clearly, we have to find P(KKA)

Now, $P(K) = \frac{4}{52}$

Also, P(K | K) is the probability of second king with the condition that one king has already been drawn. Now there are three kings in (52 – 1) = 51 cards.

Therefore, $P(K \mid K) = \frac{3}{51}$

Lastly, P(A|KK) is the probability of third drawn card to be an ace, with the condition that two kings have already been drawn. Now there are four aces in left 50 cards.

Therefore, $P(A \mid KK) = \frac{4}{50}$

By multiplication law of probability, we have,

$P(KKA) = P(K)\cdot P(K|K)\cdot P(A \mid KK)$

$= \frac{4}{52}\times\frac{3}{51}\times\frac{4}{50}$

$= \frac{2}{5525}$

12.

Here, $2P(A) = P(B) = \frac{5}{13}$;

$2P(A) = \frac{5}{13}$

$P(B) = \frac{5}{13}$

$\therefore \quad P(A) = \frac{5}{26}$

$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

$\therefore \quad P(A \cap B) = P(A \mid B) \times P(B)$

$= \frac{2}{5} \times \frac{5}{13}$

$= \frac{2}{13}$

$\therefore \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{5}{26} + \frac{5}{13} - \frac{2}{13}$

$= \frac{5}{26} + \frac{3}{13}$

$= \frac{11}{26}$

SECTION B

13.

[x] is greatest integer less than or equal to x.

Take $x_1 = 3.7$ and $x_2 = 3.2$

$f(x_1) = f(3.7) = [3.7] = 3$ | $f(x_2) = f(3.2) = [3.2] = 3$

Here, $x_1 \neq x_2$ but $f(x_1) = f(x_2)$

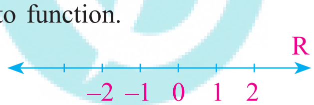
$\therefore$ Function f is not one-one function.

Here $f : R \to R$, Range of $f(x) = [x]$ is Z.

$\therefore R_f = Z \neq$ co-domain

$\therefore f$ is not onto function.

Calculation :



....

$-2 \leq x < -1 \Rightarrow [x] = -2$

$-1 \leq x < 0 \Rightarrow [x] = -1$

$0 \leq x < 1 \Rightarrow [x] = 0$

$1 \leq x < 2 \Rightarrow [x] = 1$

$2 \leq x < 3 \Rightarrow [x] = 2$ .....

$\therefore R_f = \{..., -2, -1, 0, 1, 2, ...\} = Z$

14.

Trust invest total is ₹ 30,000.

Let a trust invests ₹ x in 5% interest per year and remaining fund ₹ $(30000 - x)$ in 7% interest per year.

(a) The trust fund must obtain an annual total interest ₹ 1,800

$5\% = \frac{5}{100}$, $7\% = \frac{7}{100}$

$\therefore [x \quad 30000 - x]\begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = [1800]$

$\therefore \left[\frac{5}{100}x + \frac{7}{100}(30000 - x)\right] = [1800]$

$\therefore \left[\frac{5x + 210000 - 7x}{100}\right] = [1800]$

$-2x + 210000 = 180000$

$\therefore 2x = 30000$

$\therefore x = ₹\ 15000$

Thus, to get annual interest ₹ 1,800, first bond invests ₹ 15,000 and Second bond invests (30000 – 15000) = ₹ 15,000

(b) The trust fund must obtain an annual total interest ₹ 2,000.

$\therefore [x \quad 30000 - x]\begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = [2000]$

$\therefore \left[\frac{5x}{100} + \frac{7(30000 - x)}{100}\right] = [2000]$

$\therefore \frac{5x + 210000 - 7x}{100} = 2000$

$\therefore -2x + 210000 = 200000$

$\therefore -2x = -10000$

$\therefore x = ₹\ 5,000.$

Thus, to get annual interest ₹ 2,000, first bond invests ₹ 5,000 and Second bond invests (30000 – 5000) = ₹ 25,000

15.

$A^2 = A \cdot A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix}$

$\therefore A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

L.H.S. $= A^2 - 5A + 7I$

$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} + \begin{bmatrix} -15 & -5 \\ 5 & -10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$

$=$ R.H.S.

$A^2 - 5A + 7I = O$

Multiplying both sides by A^{-1},

$(AA)A^{-1} - 5AA^{-1} + 7IA^{-1} = OA^{-1}$

$\therefore$ A – 5I + 7A^{-1} = O

$\therefore$ 7A^{-1} = 5I – A

$$= 5\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} - \begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix}$$

$$= \begin{bmatrix}5 & 0\\0 & 5\end{bmatrix} - \begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix}$$

$$= \begin{bmatrix}5 & 0\\0 & 5\end{bmatrix} + \begin{bmatrix}-3 & -1\\1 & -2\end{bmatrix}$$

$$\therefore 7A^{-1} = \begin{bmatrix}2 & -1\\1 & 3\end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix}\frac{2}{7} & -\frac{1}{7}\\\frac{1}{7} & \frac{3}{7}\end{bmatrix}$$

16.

Suppose, $x = \sin\theta$

$$\theta = \sin^{-1}x,\ \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\therefore y = \sin^{-1}\left(2\sin\theta\sqrt{1-\sin^2\theta}\right)$$

$$= \sin^{-1}(2\sin\theta \cdot \cos\theta)$$

$$\therefore y = \sin^{-1}(\sin 2\theta)$$

$\rightarrow$ Here, $\frac{-1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$

$$\sin\left(\frac{-\pi}{4}\right) < \sin\theta < \sin\frac{\pi}{4}$$

$$\therefore \frac{-\pi}{4} < \theta < \frac{\pi}{4}$$

$$\therefore \frac{-\pi}{2} < 2\theta < \frac{\pi}{2}$$

$$\therefore 2\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \subset \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \quad \text{... (1)}$$

$y = \sin^{-1}(\sin 2\theta)$

$= 2\theta$ ($\because$ From equation (1))

$\rightarrow$ $y = 2\sin^{-1}x$

Differentiating w.r.t. x,

$$\frac{dy}{dx} = 2\frac{d}{dx}\sin^{-1}x$$

$$\therefore \frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}$$

17.

$y = \log(1+x) - \frac{2x}{2+x}$, $x > -1$

$$\frac{dy}{dx} = \frac{1}{1+x} - \left[\frac{(2+x)(2)-(2x)}{(2+x)^2}\right]$$

$$= \frac{1}{1+x} - \left[\frac{4+2x-2x}{(2+x)^2}\right]$$

$$= \frac{1}{1+x} - \frac{4}{(2+x)^2}$$

$$= \frac{(2+x)^2 - 4(1+x)}{(1+x)(2+x)^2}$$

$$= \frac{4+4x+x^2-4-4x}{(1+x)(2+x)^2}$$

$$\frac{dy}{dx} = \frac{x^2}{(1+x)(2+x)^2}$$

Now, $x > -1 \Rightarrow x^2 \geq 0$

$\Rightarrow (1+x) > 0$

$\Rightarrow (2+x)^2 > 0$

$\Rightarrow \frac{dy}{dx} \geq 0$

Therefore, $y = \log(1+x) - \frac{2x}{x+2}$,

$\therefore x > -1$ is an increasing function throughout its domain.

18.

A(1, –2, –8), B(5, 0, –2) and C(11, 3, 7)

Position vector of A = A($\vec{a}$) = $\hat{i} - 2\hat{j} - 8\hat{k}$

Position vector of B = B($\vec{b}$) = $5\hat{i} + 0\hat{j} - 2\hat{k}$

Position vector of C = C($\vec{c}$) = $11\hat{i} + 3\hat{j} + 7\hat{k}$

$\overrightarrow{AB}$ = Position vector of B – Position vector of A

$= 4\hat{i} + 2\hat{j} + 6\hat{k}$

and

$\overrightarrow{AC}$ = Position vector of C – Position vector of A

$= 10\hat{i} + 5\hat{j} + 15\hat{k}$

Here, $\frac{4}{10} = \frac{2}{5} = \frac{6}{15}$

$\therefore$ $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are collinear

$\therefore$ A, B and C are collinear.

Suppose, B divides AC from point A with the ratio of λ.

Position vector of B

$$= \frac{\lambda(\text{Position vector of C}) + (\text{Position vector of A})}{\lambda + 1}$$

$$\therefore 5\hat{i} - 2\hat{k} = \frac{\lambda(11\hat{i}+3\hat{j}+7\hat{k}) + \hat{i} - 2\hat{j} - 8\hat{k}}{\lambda+1}$$

$$\therefore 5\hat{i} - 2\hat{k} = \frac{(11\lambda+1)\hat{i}}{\lambda+1} + \frac{(3\lambda-2)\hat{j}}{\lambda+1} + \frac{(7\lambda-8)\hat{k}}{\lambda+1}$$

$$\therefore \frac{3\lambda-2}{\lambda+1} = 0$$

$$\therefore \lambda = \frac{2}{3}$$

Point B divides AC with the ratio of 2 : 3

19.

$$\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$$

$\therefore$ L : $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-3\hat{i} + 2k\hat{j} + 2\hat{k}), \lambda \in R$

$\vec{b_1} = -3\hat{i} + 2k\hat{j} + 2\hat{k}$

But, $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$

$\therefore$ M : $\vec{r} = (\hat{i} + \hat{j} + 6\hat{k}) + \mu(3k\hat{i} + \hat{j} - 5\hat{k}), \mu \in R$

$\vec{b_2} = 3k\hat{i} + \hat{j} - 5\hat{k}$

Both lines are perpendicular,

$\therefore \vec{b_1} \cdot \vec{b_2} = 0$

$\therefore (-3\hat{i} + 2k\hat{j} + 2\hat{k}) \cdot (3k\hat{i} + \hat{j} - 5\hat{k}) = 0$

$\therefore -9k + 2k - 10 = 0$

$\therefore -7k = 10$

$\therefore k = \frac{-10}{7}$

20.

$x + 2y \leq 120$

$x + y \geq 60$

$x - 2y \geq 0$

$x \geq 0$

$y \geq 0$

Objective function Z = $5x + 10y$

$x + 2y = 120$ ... (i) $\quad x + y = 60$ ... (ii)

x	0	120	
y	60	0	

(0, 60) ×
(120, 0) ✓

x	0	60
y	60	0

(0, 60) ×
(60, 0) ✓

$x - 2y = 0$ ... (iii)

x	0	2
y	0	0

(0, 0) ×
(2, 1)

Solving equation (i) and (ii),

$$\begin{array}{r} x + 2y = 120 \\ x + y = 60 \\ - \quad - \quad - \\ \hline y = 60 \end{array} \quad (0, 60) \times$$

$\therefore x = 0$

Solving equation (ii) and (iii),

$$\begin{array}{r} x + y = 60 \\ x + 2y = 0 \\ - \quad + \quad - \\ \hline 3y = 60 \end{array} \quad (40, 20) ✓$$

$\therefore y = 20$

Put y = 20 in eq^n (iii)

$x - 2(20) = 0$

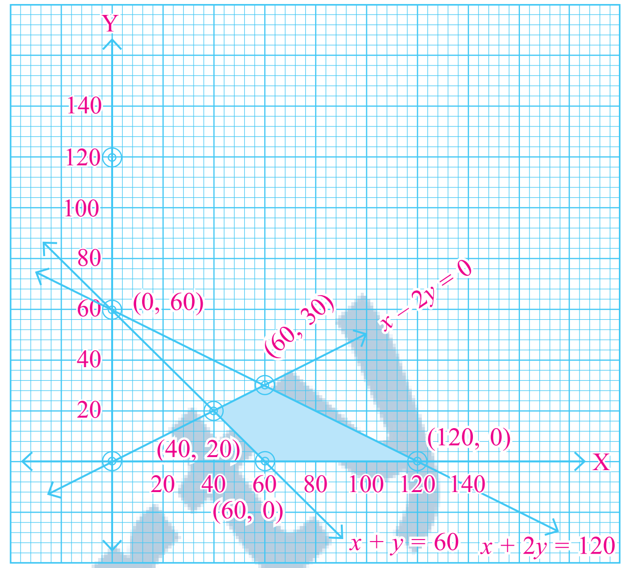
$\therefore x = 40$

Solving equation (i) and (iii),

$\therefore 4y = 120$

$\therefore y = 30 \qquad \therefore x = 60$

(60, 30) ✓ $\qquad$ (0, 0) ×



The shaded region in fig. is feasible region determined by the system of constrints which is bounded. The co-ordinates of corner points are (40, 20), (60, 30), (60, 0) and (120, 0).

Corner Point	Corresponding value of Z = $5x + 10y$
(60, 30)	600 ← Maximum
(40, 20)	400
(60, 0)	300 ← Minimum
(120, 0)	600 ← Maximum

Thus, The maximum value of Z is 600 and Minimum value of Z is 300.

21.

60% students reside in a hostel

Event A : A student is hostelier

$$P(A) = \frac{60}{100}$$

40% are day scholars.

Event B : Student is not living in hostel.

$$P(B) = \frac{40}{100}$$

Event E : Student get A grade

$$P(E) = P(A) \cdot P(E \mid A) + P(B) \cdot P(E \mid B)$$

$$= \frac{60}{100} \times \frac{30}{100} + \frac{40}{100} \times \frac{20}{100}$$

$$= \frac{18}{100} + \frac{8}{100}$$

$$= \frac{26}{100}$$

$$P(A \mid E) = \frac{P(A) \cdot P(E|A)}{P(E)}$$

$$= \frac{\frac{60}{100} \times \frac{30}{100}}{\frac{26}{100}}$$

$$= \frac{9}{13}$$

SECTION C

22.

Since A, B, C are all square matrices of order 2, and CD – AB is well defined, D must be a square matrix of order 2.

Let $D = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Then CD – AB = O gives

$$\therefore \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix} = O$$

or $$\begin{bmatrix} 2a+5c & 2b+5d \\ 3a+8c & 3b+8d \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

or $$\begin{bmatrix} 2a+5c-3 & 2b+5d \\ 3a+8c-43 & 3b+8d-22 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

By equality of matrices, we get

$2a + 5c - 3 = 0$ .... (1)

$3a + 8c - 43 = 0$ .... (2)

$2b + 5d = 0$ .... (3)

and $3b + 8d - 22 = 0$ .... (4)

Solving (1) and (2),

we get $a = -191$, $c = 77$.

Solving (3) and (4),

we get $b = -110$, $d = 44$.

Therefore $D = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$= \begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix}.$$

23.

The equation can be represented as matrix form,

$$\therefore \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

$\therefore$ AX = B

Where, $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$

AX = B

$\therefore X = A^{-1}B$

For finding A^{-1},

$$|A| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{vmatrix}$$

$= 1(12 - 5) + 1(9 + 10) + 2(-3 - 8)$

$= 7 + 19 + 2(-11)$

$= 26 - 22$

$= 4 \neq 0$

$\therefore$ We get unique solution.

For finding *adj* A,

Co-factor of element 1 $A_{11} = (-1)^2 \begin{vmatrix} 4 & -5 \\ -1 & 3 \end{vmatrix}$

$= 1(12 - 5)$

$= 7$

Co-factor of element –1 $A_{12} = (-1)^3 \begin{vmatrix} 3 & -5 \\ 2 & 3 \end{vmatrix}$

$= (-1)(9 + 10)$

$= -19$

Co-factor of element 2 $A_{13} = (-1)^4 \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix}$

$= 1(-3 - 8)$

$= -11$

Co-factor of element 3 $A_{21} = (-1)^3 \begin{vmatrix} -1 & 2 \\ -1 & 3 \end{vmatrix}$

$= (-1)(-3 + 2)$

$= 1$

Co-factor of element 4 $A_{22} = (-1)^4 \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix}$

$= 1(3 - 4)$

$= -1$

Co-factor of element –5 $A_{23} = (-1)^5 \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix}$

$= (-1)(-1 + 2)$

$= -1$

Co-factor of element 2 $A_{31} = (-1)^4 \begin{vmatrix} -1 & 2 \\ 4 & -5 \end{vmatrix}$

$= 1(5 - 8)$

$= -3$

Co-factor of element –1 $A_{32} = (-1)^5 \begin{vmatrix} 1 & 2 \\ 3 & -5 \end{vmatrix}$

$= (-1)(-5 - 6)$

$= 11$

Co-factor of element 3 $A_{33} = (-1)^6 \begin{vmatrix} 1 & -1 \\ 3 & 4 \end{vmatrix}$

$= 1(4 + 3)$

$= 7$

$$adj\ A = \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\ adj\ A$$

$$\therefore\ A^{-1} = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$$

➪ $X = A^{-1}B$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}\begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 49-5-36 \\ -133+5+132 \\ -77+5+84 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

Solution : $x = 2,\ y = 1,\ z = 3$

24.

➪ $y = Ae^{mx} + Be^{nx}$

Differentiate w.r.t. x,

$$\frac{dy}{dx} = Ae^{mx} \cdot m + Be^{nx} \cdot n$$

$$\therefore \frac{dy}{dx} = m \cdot Ae^{mx} + n \cdot Be^{nx} \qquad \text{........ (1)}$$

Now, Differentiate again w.r.t. x,

$$\frac{d^2y}{dx^2} = Ae^{mx} \cdot m^2 + Be^{nx} \cdot n^2$$

$$\therefore \frac{d^2y}{dx^2} = m^2 \cdot Ae^{mx} + n^2 \cdot Be^{nx} \qquad \text{........ (2)}$$

$$\text{L.H.S.} = \frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny$$

$$= [m^2\ Ae^{mx} + n^2\ Be^{nx}]$$

$$- (m+n)\ (m\ Ae^{mx} + n\ Be^{nx})$$

$$+ mn\ (Ae^{mx} + Be^{nx})$$

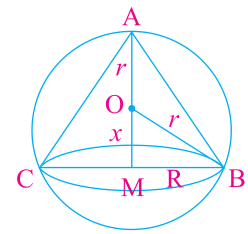
$$= m^2\ Ae^{mx} + n^2\ Be^{nx} - m^2\ Ae^{mx} - mn\ Be^{nx}$$

$$- mn\ Ae^{mx} - n^2\ Be^{nx}$$

$$+ mn\ Ae^{mx} + mn\ Be^{nx}$$

$$= 0 = \text{R.H.S.}$$

25.



➪ Here, radius of sphere is r.

Suppose, radius and height of cone is R and h respectively.

$\therefore$ MB = R

MA = $h = x + r$

From the fig., In ΔOBM, $r^2 = x^2 + R^2$ ..... (1)

$\rightarrow$ Volume of cone (V) $= \frac{1}{3}\pi R^2 h$

$$= \frac{1}{3}(\pi)(r^2 - x^2)(r + x)$$

($\because$ From equation (1))

$$= \frac{1}{3}(\pi)(r^3 + r^2x - x^2r - x^3)$$

$$\therefore\ f(x) = \frac{\pi}{3}(r^3 + r^2x - x^2r - x^3)$$

$$\therefore\ f'(x) = \frac{\pi}{3}(0 + r^2 - 2xr - 3x^2)$$

$$\therefore\ f''(x) = \frac{\pi}{3}(0 + 0 - 2r - 6x)$$

$$= \frac{-2\pi}{3}(r + 3x) < 0$$

$\rightarrow$ For finding maximum volume,

$$f'(x) = 0$$

$$\therefore\ \frac{\pi}{3}(r^2 - 2xr - 3x^2) = 0$$

$$\therefore\ r^2 - 2xr - 3x^2 = 0$$

$$\therefore\ r^2 - 3xr + xr - 3x^2 = 0$$

$$\therefore\ r(r - 3x) + x(r - 3x) = 0$$

$$\therefore\ (r - 3x)(x + r) = 0$$

$\therefore\ r - 3x = 0$	$x + r = 0$
$\therefore\ r = 3x$	$r = -x$
$\therefore\ x = \frac{r}{3}$	$x = -r$ which is not possible ($\because x > 0$)

$\rightarrow$ Height of cone (h) $= x + r$

$$= r + \frac{r}{3}$$

$$\therefore\ h = \frac{4r}{3}$$

26.

➪ Suppose, $I = \int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x}\,dx$

By property (6),

$$I = \int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1 + \cos^2(\pi - x)}\,dx \quad \{¤u.$$

$$\therefore\ I = \int_0^{\pi} \frac{(\pi - x)\sin x}{1+\cos^2 x}\,dx$$

$$= \pi \int_0^{\pi} \frac{\sin x\,dx}{1+\cos^2 x} - I$$

$$2I = \pi \int_0^{\pi} \frac{\sin x}{1+\cos^2 x}\,dx$$

$$I = \frac{\pi}{2} \int_0^{\pi} \frac{\sin x}{1+\cos^2 x}\,dx$$

Take, $\cos x = t$

So that $-\sin x\,dx = dt$.

$x = 0 \Rightarrow t = 1$ and

$x = \pi \Rightarrow t = -1$.

$$\therefore\ I = \frac{-\pi}{2} \int_1^{-1} \frac{dt}{1+t^2}$$

$$= \frac{\pi}{2} \int_{-1}^{1} \frac{dt}{1+t^2}$$

$$= \pi \int_0^{1} \frac{dt}{1+t^2}$$

(Property (8)(i), $\frac{1}{1+t^2}$ is even function)

$$= \pi\left[\tan^{-1} t\right]_0^1$$

$$= \pi[\tan^{-1} 1 - \tan^{-1} 0]$$

$$= \pi\left[\frac{\pi}{4} - 0\right]$$

$$= \frac{\pi^2}{4}$$

27.

➩ $$\frac{dy}{dx} + \frac{y^2+y+1}{x^2+x+1} = 0$$

$$\therefore\ \frac{dy}{dx} = -\frac{(y^2+y+1)}{x^2+x+1}$$

$$\therefore\ \frac{dy}{y^2+y+1} = -\frac{dx}{x^2+x+1}$$

→ Integrate both the sides,

$$\therefore\ \int \frac{dy}{y^2+y+1} = -\int \frac{dx}{x^2+x+1}$$

$$\therefore\ \int \frac{dy}{y^2+2\left(\frac{y}{2}\right)+\frac{1}{4}-\frac{1}{4}+1} = -\int \frac{dx}{x^2+2\left(\frac{x}{2}\right)+\frac{1}{4}-\frac{1}{4}+1}$$

$$\therefore\ \int \frac{dy}{\left(y+\frac{1}{2}\right)^2+\frac{3}{4}} = -\int \frac{dx}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}}$$

$$\therefore\ \frac{1}{\frac{\sqrt{3}}{2}} \tan^{-1}\left[\frac{\left(y+\frac{1}{2}\right)}{\frac{\sqrt{3}}{2}}\right] = -\frac{1}{\frac{\sqrt{3}}{2}} \tan^{-1}\left[\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right] + c$$

$$\therefore\ \frac{2}{\sqrt{3}} \tan^{-1}\left[\frac{2y+1}{\sqrt{3}}\right] = -\frac{2}{\sqrt{3}} \tan^{-1}\left[\frac{2x+1}{\sqrt{3}}\right] + c$$

$$\therefore\ \tan^{-1}\left[\frac{2y+1}{\sqrt{3}}\right] + \tan^{-1}\left[\frac{2x+1}{\sqrt{3}}\right] = \frac{\sqrt{3}}{2}\ c$$

$$\therefore\ \tan^{-1}\left[\frac{\left(\frac{2y+1}{\sqrt{3}}\right)+\left(\frac{2x+1}{\sqrt{3}}\right)}{1-\left(\frac{2y+1}{\sqrt{3}}\right)\left(\frac{2x+1}{\sqrt{3}}\right)}\right] = \frac{\sqrt{3}}{2}\ c$$

$$\therefore\ \frac{(2y+1)+(2x+1)}{\sqrt{3}} \times \frac{3}{3-(2y+1)(2x+1)} = \tan\left[\frac{\sqrt{3}}{2}c\right]$$

$$\therefore\ \frac{\sqrt{3}\,[2x+2y+2]}{3-(4xy+2y+2x+1)} = \tan\left[\frac{\sqrt{3}}{2}c\right]$$

$$\therefore\ \frac{2\sqrt{3}\,(x+y+1)}{2-2x-2y-4xy} = \tan\left[\frac{\sqrt{3}}{2}c\right]$$

$$\therefore\ \frac{2\sqrt{3}\,(x+y+1)}{2(1-x-y-2xy)} = \tan\left[\frac{\sqrt{3}}{2}c\right]$$

$$\therefore\ (x+y+1) = \frac{1}{\sqrt{3}} \tan\left[\frac{\sqrt{3}}{2}c\right](1-x-y-2xy)$$

$$\therefore\ (x+y+1) = A(1-x-y-2xy)$$

Where, $A = \frac{1}{\sqrt{3}} \tan\left[\frac{\sqrt{3}}{2}c\right]$ is arbitary const.